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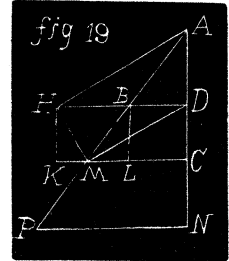
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NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

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[Continued from the June Number.]

PROPOSITION XX. *Let there be a triangle ACM (fig. 19.) right-angled at C . Then from the point B bisecting this AM let fall the perpendicular BD to AC . I say this perpendicular will not be (in the hypothesis of acute angle) greater than half the perpendicular MC .*



Proof. For let DB be produced to DH double DB . Therefore DH would be (if DB be greater than the aforesaid half) greater than CM , and therefore equal to a certain continuation CMK .

Join AH, HK, HM, MD .

Now we proceed thus. Since in the triangles HBA, DBM , the sides HB, BA are assumed equal to the sides DB, BM ; and (Eu. I. 15) the angles at the point B are equal; also (Eu. I. 4) the base HA will be equal to the base MD .

Then, by the same reasoning, in the triangles HBM, DBA , the bases HM, DA will be equal.

Wherefore in the triangles MHA, ADM , (Eu. I. 8) the angles MHA, ADM will be equal. Again in the triangles AHB, MDB , the residual angle MHB will remain equal to the residual right angle ADB . Therefore the angle MHB will be right. But this is absurd in the hypothesis of acute angle; since the straight KH joining equal perpendiculars KC, HD , makes (P. III.) acute angles with these perpendiculars.

Therefore the perpendicular BD is not (in the hypothesis of acute angle) greater than the half of the perpendicular MC : Quod erat demonstrandum.

PROPOSITION XXI. *The same remaining; if AM , and AC are understood as produced in infinitum. I say their distance (in either hypothesis, of right angle or of acute angle) will be greater than any assignable finite length.*

Proof. In AM produced assume AP double AM , and let fall to AC produced the perpendicular PN .

The perpendicular MC will not be in either of the aforesaid hypothesis greater than half the perpendicular PN .

Therefore PN will be at least double MC , just as MC is at least double BD . And so always, if in AM produced is assumed double AP , and from the termination of this a perpendicular is let fall to AC produced.

It is obvious the perpendicular, which from AM ever more produced is let fall to AC produced, will be a multiple of the determinate BD beyond any finite assignable number.

Therefore the distance of the aforesaid straightness will be (in either aforesaid hypothesis) greater than any assignable finite length.

Quod erat demonstrandum,